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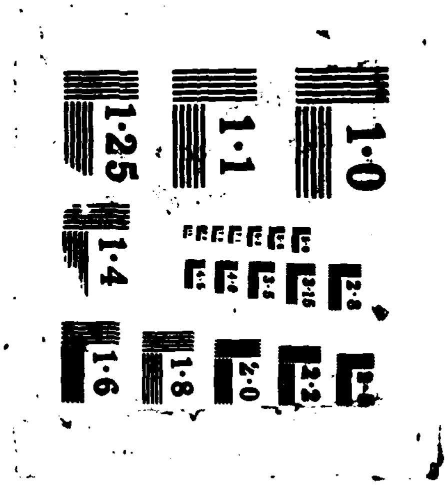
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TECHNICAL REPORT ARCCB-TR-87020

STOCHASTIC ANALYSIS FOR NAVIGATION OF AUTONOMOUS PLATFORMS USING RANGE FINDERS

C. N. SHEN

AUGUST 1987

US ARMY ARMAMENT RESEARCH, DEVELOPMENT
AND ENGINEERING CENTER
CLOSE COMBAT ARMAMENTS CENTER
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

For range finders having noncooperative targets and considerable measurement errors, a stochastic analysis is necessary to determine the differentials such as the gradient of a terrain during navigation of an autonomous platform. The angular measurement errors in elevation or azimuth contribute a deteriorated effect of the gradient estimate, especially when the terrain or target is far away. The smoothing of these gradients can be obtained by using an optimization method for approximation involving spline functions. This method

(CONT'D ON REVERSE)

20. ABSTRACT (CONT'D)

can be applied to solve the following problems: whether the platform can climb on the estimated in-path slope or whether it will tip over on the estimated cross-path.

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PROBLEM DESCRIPTION

The problem presented here is part of the investigations conducted for the design of the data acquisition and decision making system for an unmanned exploratory platform. As we know, it is necessary for the platform to be equipped with a data acquisition and a decision making system.

Usually, a laser range finder is chosen as a main sensing device which can determine the distance to a point within a certain accuracy. With reference to Figure 1, the terrain is scanned by varying the azimuth angle, θ_i, and the elevation angle, β_i, of the laser beam in a discrete fashion. In general, the measurements are then available in the form of a "range matrix" as

$$M = \{m_{ij}\} \tag{1}$$

where $i = 1,2,\ldots,N$, correspond to the elevation angle, β_i

$j = 1,2,\ldots,L$, correspond to the azimuth angle, θ_j

The range matrix M above describes a certain scanned area of the ground in front of the mobile robot platform and can be used to detect edges of discrete objects or to estimate the slopes of the terrain. This is a two-dimensional grid. In our study for simplicity, a one-dimensional case is first considered, that is, change β_i for a fixed θ_j. Thus, we have observation data m_i corresponding to β_i, from which we can estimate the in-path slopes of the range.

Generally speaking, both β_i and m_i are subject to random errors such that

$$\beta_i = \xi_i + v_i \tag{2}$$

$$m_i(\beta_i) = f(\beta_i) + \mu_i \tag{3}$$

where

ξ_i are known knots; (4a)

$f(\xi)$ is an unknown function; (4b)

β_i are stochastic intervals; (5a)

$m_i(\beta_i)$ are measurements corrupted by noise; (5b)

v_i and μ_i are random errors with zero means, for all $i = 1,2,...N$.

Equations (2) and (3) imply that both independent and dependent variables are corrupted by random noise, which may make the problem more complicated, as shown in Figure 2.

With the nodal described above, we are now able to formulate the problem.

OBJECTIVE FUNCTION FORMULATION

Referring to Figure 3, it is apparent that we cannot use the method of interpolation directly to approximate the unknown function f since the β_i are stochastic. Instead of this, we would like to estimate f by an approximating function h; our objective is to determine the values of function h and its first derivatives due to random angle error v_i.

The following relations can be derived by using Taylor's series expansion at β_i:

$$f(\beta_i) = f(\xi_i) + \frac{df(\xi_i)}{d\xi} v_i + o(v_i) \tag{6}$$

where o(v) represents the higher order term of v. Using the first-order approximation yields:

$$f(\beta_i) \cong f(\xi_i) + \frac{df(\xi_i)}{d\xi} v_i \tag{7}$$

Similarly, for the approximating function h to estimate the unknown function f, we have

$$h(\beta_i) \overset{\Delta}{=} h(\xi_i) + \frac{dh(\xi_i)}{d\xi} v_i \tag{8}$$

From the equations above, one obtains

$$\lambda_i \overset{\Delta}{=} h(\beta_i) - h(\xi_i) = \frac{dh(\xi_i)}{d\xi} (v_i) \tag{9}$$

The stochastic difference λ_i for the approximating function h should be as small as possible in this problem.

We will attempt to determine a function approximation h with a compromise between the following objectives:

1. The measurement value m_i should be fitted close enough.

2. The approximating function h should be smooth enough in the sense that the discontinuities in its second derivative are as small as possible.

Now in order to be able to formulate this criterion mathematically, we need to find some measure of smoothness and some measure of closeness of fit. To be more specific, suppose that L is a linear space of "smooth" functions and that J_c is a functional defined on L which measures how well a function fits the data. Suppose in addition that J_s is a functional defined on L which measures the smoothness of an element on L, then the smoothing problem is the following: find $s \in L$, such that

$$J(s) = \inf_{h \in L} J(h) \text{ or } J(s) \leq J(h) \tag{10}$$

where

$$J(h) = J_c(h) + J_s(h) \tag{11}$$

<u>**The Measure of Closeness**</u>

From Figure 3, in order to fit data (v_i, m_i) as closely as possible, we can simply take the sum of squared errors from Eqs. (3) and (9) as a measure of closeness of fit J_c, that is

$$J_c = \sum_{i=1}^{N} (v_i \frac{dh}{d\xi})^T \hat{R}_i^{-1} (v_i \frac{dh}{d\xi}) +$$

$$+ \sum_{i=1}^{N} [h(\xi_i) - m_i(\xi_i)]^T R_i^{-1} [h(\xi_i) - m_i(\xi_i)] \tag{12}$$

where from Eq. (3)

$$R_i \overset{\Delta}{=} E[\mu_i \mu_i^T] \tag{13}$$

Here, R_i^{-1} and $\hat{R}_i^{-1}$ are weighing matrices which reflect our degree of confidence in the data m_i and v_i. For convenience, it is assumed that R_i and $\hat{R}_i$ are constants for all $i = 1,2,\ldots,N$.

From Eq. (9), the error λ_i due to the random noise v_i can be written as

$$\lambda_i = \frac{dh(\xi_i)}{d\xi} v_i \tag{14}$$

It is assumed that the random noise v_i is a certain stochastic process with zero mean as studied in the next section. Thus, the error covariance matrix for λ_i is

$$E[\lambda_i \lambda_i^T] = \frac{dh(\xi_i)}{d\xi} E[v_i v_i^T]\left[\frac{dh(\xi_i)}{d\xi}\right]^T \tag{15}$$

<u>The Measure of Smoothness</u>

From the viewpoint of the approximation theory, the spline function provides a means of optimally reconstructing an unknown function f such that

$$\int_{\xi_1}^{\xi_N} [s^2(\xi)]^2 d\xi \leq \int_{\xi_1}^{\xi_N} [h^2(\xi)]^2 d\xi = J_s \tag{16}$$

where s is a spline function, $s \in C^2$, and h is an approximation function of the unknown function f and $h \in C^2$. Thus, $s(\xi)$ is the function, among those satisfying some constraints, that is "smoothest" is the sense of Eq. (16).

To be more convenient and simpler, in our study we will use the piecewise cubic spline $s \in C^2$. Then Eq. (16) can be rewritten as

$$J_s = \rho \sum_{i=1}^{N} \int_{\xi_{i-1}}^{\xi_i} [h(\xi)]^2 d\xi \tag{17}$$

In summary, substituting Eqs. (12) and (17) into Eq. (11) yields an objective functional J

$$J = \sum_{i=1}^{N} [h(\xi_i) - m_i]^T R_i^{-1} [h(\xi_i) - m_i]$$

$$+ \sum_{i=1}^{N} \left(\frac{dh}{d\xi} v_i\right)^T \hat{R}_i^{-1} \left(\frac{dh}{d\xi} v_i\right) + \rho \sum_{i=1}^{N} \int_{\xi_{i-1}}^{\xi_i} [\ddot{h}(\xi)]^2 d\xi \tag{18}$$

where $\rho > 0$ is a smoothing parameter.

STOCHASTIC EXPECTATION

Since v_i is a stochastic variable, we use the expected value of J, E[J], as the functional to be minimized, i.e.,

$$E[J] = E\Big\{ \sum_{i=1}^{N} [h(\xi_i) - m_i]^T R_i^{-1} [h(\xi_i) - m_i]$$

$$+ \sum_{i=1}^{N} \left(\frac{dh}{d\xi} v_i\right)^T \hat{R}_i^{-1} \left(\frac{dh}{d\xi} v_i\right) + \rho \sum_{i=2}^{N} \int_{\xi_{i-1}}^{\xi_i} [\ddot{h}(\xi)]^2 d\xi \Big\} \tag{19}$$

where h is a set of any piecewise cubic Hermite polynomials and $h \in C^1$.

In order to obtain the solution to Eq. (19), the following algebraic manipulations for Eq. (19) are necessary:

$$\text{Define } h_i = h(\xi_i) \quad , \quad \dot{h}_i = \dot{h}(\xi_i)$$

$$E[J] = \sum_{i=1}^{N} [(h_i - m_i)^T R_i^{-1} (h_i - m_i) + E(v_i^T \dot{h}_i^T \hat{R}_i^{-1} \dot{h}_i v_i)] + \rho \sum_{i=2}^{N} \int [\ddot{h}(\xi)]^2 d\xi \tag{20}$$

Using the matrix identity, the second term of the above equation becomes

$$E(v_i^T \dot{h}_i^T \hat{R}_i^{-1} \dot{h}_i v_i) = \text{Trace } [(\dot{h}_i \hat{R}_i^{-1} \dot{h}_i) E(v_i v_i^T)]$$

and

$$E(v_i v_i^T) \stackrel{\Delta}{=} \sigma^2 \tag{21}$$

Then,

$$E[J] = \sum_{i=1}^{N} (h_i-m_i)^T R_i^{-1} (h_i-m_i) + \mathrm{Trace}[(\dot{h}_i^T \hat{R}_i^{-1} \dot{h}_i) E(v_i v_i^T)]$$

$$= \sum_{i=1}^{N} [(h_i-m_i)^T R_i^{-1} (h_i-m_i) + (\sigma_i \dot{h}_i)^T \hat{R}_i^{-1} (\sigma_i \dot{h}_i)] + \rho \sum_{i=2}^{N} \int_{\xi_{i-1}}^{\xi_i} [\ddot{h}(\xi)]^2 d\xi \qquad (22)$$

As mentioned previously, our main purpose is to obtain the estimates of the values of the unknown function and its first derivatives at grids ξ_i, $i=1,2,\ldots,N$, i.e., $h(\xi_i)$ and $\ddot{h}(\xi_i)$. To make the problem simpler, it is proposed that the state variable approach be used. Thus the optimization problem in Eq. (19) can be transformed into a state estimation problem, where each state vector x_i is defined as:

$$x_i = [h(\xi), \dot{h}(\xi)]^T \Big|_{\xi_i} \qquad (23)$$

Hence, the objective functional J becomes

$$E[J] = \sum_{i=1}^{N} [(\bar{H}x_i-m_i)^T R_i^{-1} (\bar{H}x_i-m_i) + (\tilde{H}x_i)^T \hat{R}_i^{-1} (\tilde{H}x_i)]$$

$$+ \rho \sum_{i=2}^{N} \int_{\beta_{i-1}}^{\beta_i} [\ddot{p}(\xi)]^2 d\xi \qquad (24)$$

where

$$\bar{H} = [1,0] \ , \ \tilde{H} = [0,\sigma_i] \qquad (25)$$

Now the key to solving the problem in Eq. (24) is to evaluate the smoothing integral so as to obtain an expression in terms of state variables.

<u>Problem Formulation for Two-Dimensional Grid</u>

When the observation data are noise corrupted and the underlying system is unknown, it is proposed to approximate the original signal by spline functions which minimize a certain objective function. Thus, from a set of discrete measurements, $m_{i,j}$, corrupted by the white noise process, $v_{i,j}$

$$m_{i,j}(\beta_i,\sigma_j) = f(\beta_i,\theta_j) + u_{i,j}\ , \ i = 1,2,\ldots,N \quad j = 1,2,\ldots,N \tag{26}$$

with angle errors

$$\beta_i = \xi_i + v_i \quad \text{and} \quad \theta_j = \eta_j + u_j \tag{27}$$

The original two-dimensional signal $f(\xi,\eta)$ defined in the region of (ξ,η) is approximated by a spline function $s(\xi,\eta)$ which minimizes the following objective function:

$$J = J_c + J_s \tag{28}$$

where

$$J_c = \sum_{j=1}^{M}\sum_{i=1}^{N} [s(\beta_i,\theta_j) - m_{i,j}]^T R_{i,j}^{-1}[s(\beta_i,\theta_j) - m_{i,j}]$$

$$+ \sum_{j=1}^{M}\sum_{i=1}^{N} [s(\beta_i,\theta_j) - s(\xi_i,\eta_j)]^T \hat{R}_{ij}^{-1}[s(\beta_i,\theta_j) - s(\xi_i,\eta_j)] \tag{29}$$

and

$$J_s = \rho\left[\int_{\eta_1}^{\eta_M}\int_{\xi_1}^{\xi_N} z(\xi,\eta)d\xi dn\right] \tag{30}$$

where $\rho > 0$ is the smoothing parameter; $R_{i,j} \overset{\Delta}{=} E[\mu_{ij}\mu_{ij}^T]$ is the observation error covariance; and $z(\xi,\eta)$ is a certain smoothness measure of $s(\xi,\eta)$ at (ξ,η).

The following relation can be derived by using Taylor's series expansion at (β_i,θ_j):

$$s(\beta_i,\theta_j) - s(\xi_i,\eta_j) = \frac{\partial s}{\partial \xi}(\xi_i,\eta_j)\, v_i + \frac{\partial s}{\partial \xi}(\xi_i,\eta_j)u_j + o(v_i)o(u_j) \tag{31}$$

Since v_i and u_j are stochastic variables, we will use the expected value of J_c, $E(J_c)$ as the functional to be minimized.

Define

$$s_\xi(i,j) = \frac{\partial s}{\partial \xi}(\xi_i,\eta_j) \tag{32}$$

$$s_\eta(i,j) = \frac{\partial s}{\partial \eta}(\xi_i,\eta_j) \tag{33}$$

We have

$$E[J_c] = \sum_{j=1}^{M} \sum_{i=1}^{N} \{[s(i,j)-m_{ij}]^T R_{ij}^{-1}[s(i,j)-m_{ij}]$$

$$+ [\sigma_i s_\xi(i,j)]^T \tilde{R}_{ij}^{-1}[\sigma_i s_\xi(i,j)]$$

$$+ [\Omega_j s_\eta(i,j)]^T \hat{R}_{ij}^{-1}[\Omega_j s_\eta(i,j)]\} \tag{34}$$

where

$$\sigma_i^2 = E[v_i v_i^T] \tag{35}$$

and

$$\Omega_j^2 = E[u_j u_j^T] \tag{36}$$

Choice of An Approximating Function

In this report, we are interested in obtaining smoothed estimates of function values and the first derivatives in both ξ and η directions. Here, we propose to restrict our approximating functions to piecewise bicubic Hermite polynomials which have continuous first derivatives in both ξ and η directions. Let us define

$$x_{i,j} \stackrel{\Delta}{=} \left[s(\xi,\eta) \quad , \quad \frac{\partial s}{\partial \xi}(\xi,\eta) \quad , \quad \frac{\partial s}{\partial \eta}(\xi,\eta) \quad , \quad \frac{\partial^2 s}{\partial \xi \partial \eta}(\xi,\eta)\right]^T_{(\xi_i,\eta_j)} \tag{37}$$

for $i = 1,2,\ldots,N$ and $j = 1,2,\ldots,M$. Then a piecewise bicubic Hermite polynomial is completely defined by $x_{i,j}$, $i = 1,2,\ldots,M$ as follows:

$$s(\xi,\eta) = s_{i,j}(\xi,\eta) \quad , \quad \text{for } \xi_i \leqq \xi \leqq \xi_{i+1} \tag{38}$$

and

$$\eta_j \leqq \eta \leqq \eta_{j+1} \tag{39}$$

Thus, $s_{i,j}(\xi,\eta)$ can be written in terms of the basis functions ϕ and ψ, which are the bicubic Hermite polynomials (spline functions), as follows:

$$s_{i,j}(\xi,\eta) = \sum_{\ell=0}^{1}\sum_{m=0}^{1} \begin{bmatrix} \phi_\ell(\xi)\cdot\phi_m(\eta) \\ \psi_\ell(\xi)\cdot\phi_m(\eta) \\ \phi_\ell(\xi)\cdot\psi_m(\eta) \\ \psi_\ell(\xi)\cdot\psi_m(\eta) \end{bmatrix}^T \cdot x_{i+\ell,j+m} \tag{40}$$

Then Eq. (34) becomes

$$\begin{aligned} E[J_c] = \sum_{j=1}^{M}\sum_{i=1}^{N} \{ & [Hx_{i,j} - m_{i,j}]^T R_{ij}^{-1} [Hx_{i,j} - m_{ij}] \\ & + [\tilde{H}_i x_{ij}]^T \tilde{R}_{ij}^{-1} [\tilde{H}_i x_{ij}] \\ & + [\hat{H}_j x_{ij}]^T \hat{R}_{ij}^{-1} [\hat{H}_j x_{ij}] \} \end{aligned} \tag{41}$$

where

$$H = [1\ 0\ 0\ 0] \tag{42}$$

$$\tilde{H}_i = [0\ \sigma_i\ 0\ 0] \tag{43}$$

$$\hat{H}_j = [0\ 0\ \Omega_j\ 0] \tag{44}$$

Choice of the Smoothness Measure $z(\xi,\eta)$

Here we present three examples of the smoothness measures and compare their physical implications.

1. Gaussian curvature: The mean curvature of a surface at (ξ,η) is defined as:

$$(0.5)\nabla^2 s(\xi,\eta) \tag{45}$$

Noting in Euler's theorem that the sum of two curvatures in perpendicular directions at a point is constant, the square of $\nabla^2 s(\xi,\eta)$ in Eq. (45) would be a reasonable measure for the smoothness of a surface

$$z(\xi,\eta) = [\frac{\partial^2}{\partial\xi^2} s(\xi,\eta) + \frac{\partial^2}{\partial\eta^2} s(\xi,\eta)]^2 \tag{46}$$

2. A variation from the Gaussian curvature: With reference to Eq. (46), an interesting case occurs when the two principal curvatures are equal and of the opposite sign. The mean curvature in this case is zero. This is the so-called "saddle point" and every surface element of such a membrane is "pure twist." An appropriate smoothness measure would be changed to:

$$z(\xi,\eta) = [\frac{\partial^2}{\partial \xi^2} s(\xi,\eta)]^2 + [\frac{\partial^2}{\partial \eta^2} s(\xi,\eta)]^2 \tag{47}$$

3. It is suggested to use $\| \nabla^4 s(\xi,\eta) \|^2$ as a smoothness measure for a surface. The physical interpretation of the quantity $\nabla^4 s(\xi,\eta)$ is found in a plate bending theory; an unloaded plate can bend only in a biharmonic function ω where

$$\nabla^4 \omega = 0 \tag{48}$$

<u>Smoothing Integral</u>

Now we need to determine the function $s(\xi,\eta)$ which minimizes the objective function J in Eq. (28). It is noted that the smoothing integral in its present form gives difficulties in finding an explicit solution. By evaluating the integrals of the derivatives of basis functions and applying some algebraic manipulations, these smoothing integrals are converted to quadratic forms as follows:

$$\begin{aligned} J_s &= \rho \int_{\eta_1}^{\eta_M} \int_{\epsilon_1}^{\epsilon_N} [z(\xi,\eta)] d\xi d\eta = \rho \sum_{j=1}^{M-1} \sum_{i=1}^{N-1} \int_{\eta_j}^{\eta_{j+1}} \int_{\xi_i}^{\xi_{i+1}} [z(\xi,\eta)] d\xi d\eta \\ &= \sum_{j=1}^{M-1} \sum_{i=1}^{N-1} (x_{i,j}^T, x_{i+1,j}^T, x_{i,j+1}^T, x_{i+1,j+1}^T) \cdot \\ &\qquad C \cdot (x_{i,j}^T, x_{i+1,j}^T, x_{i,j+1}^T, x_{i+1,j+1}^T)^T \end{aligned} \tag{49}$$

where C is a 16 by 16 matrix.

Now we can put the objective function together from Eqs. (28), (45), and (49) as

$$J = E[J_c] + J_s$$

$$= \sum_{j=1}^{M} \sum_{i=1}^{N} \{[Hx_{i,j} - m_{ij}]^T R_{ij}^{-1} [Hx_{i,j} - m_{ij}]$$

$$+ [\tilde{H}_i x_{ij}]^T \tilde{R}_{ij}^{-1} [\tilde{H}_i x_{ij}] + [\hat{H}_j x_{ij}]^T \hat{R}_{ij}^{-1} [\hat{H}_j x_{ij}]\}$$

$$+ \sum_{j=1}^{M-1} \sum_{i=1}^{N-1} [x_{i,j}^T, x_{i+1,j}^T, x_{i,j+1}^T, x_{i+1,j+1}^T]$$

$$\cdot C[x_{i,j}^T, x_{i+1,j}^T, x_{i,j+1}^T, x_{i+1,j+1}^T]^T \tag{50}$$

For the optimal values of J, one can take the partial derivative of J with respect to the state variables $x_{i,j}$, etc. and set to zero.

CONCLUSIONS

The effect of random angular errors to the range measurement has been studied with the approximating function at the random node modeled. The functional is expressed first in terms of the smooth functions and then in terms of state variables. An expected value of this functional is formulated so that it is in terms of the covariance of the random angular errors. The work has been extended to two-dimensional random grids using bicubic Hermite polynomials.

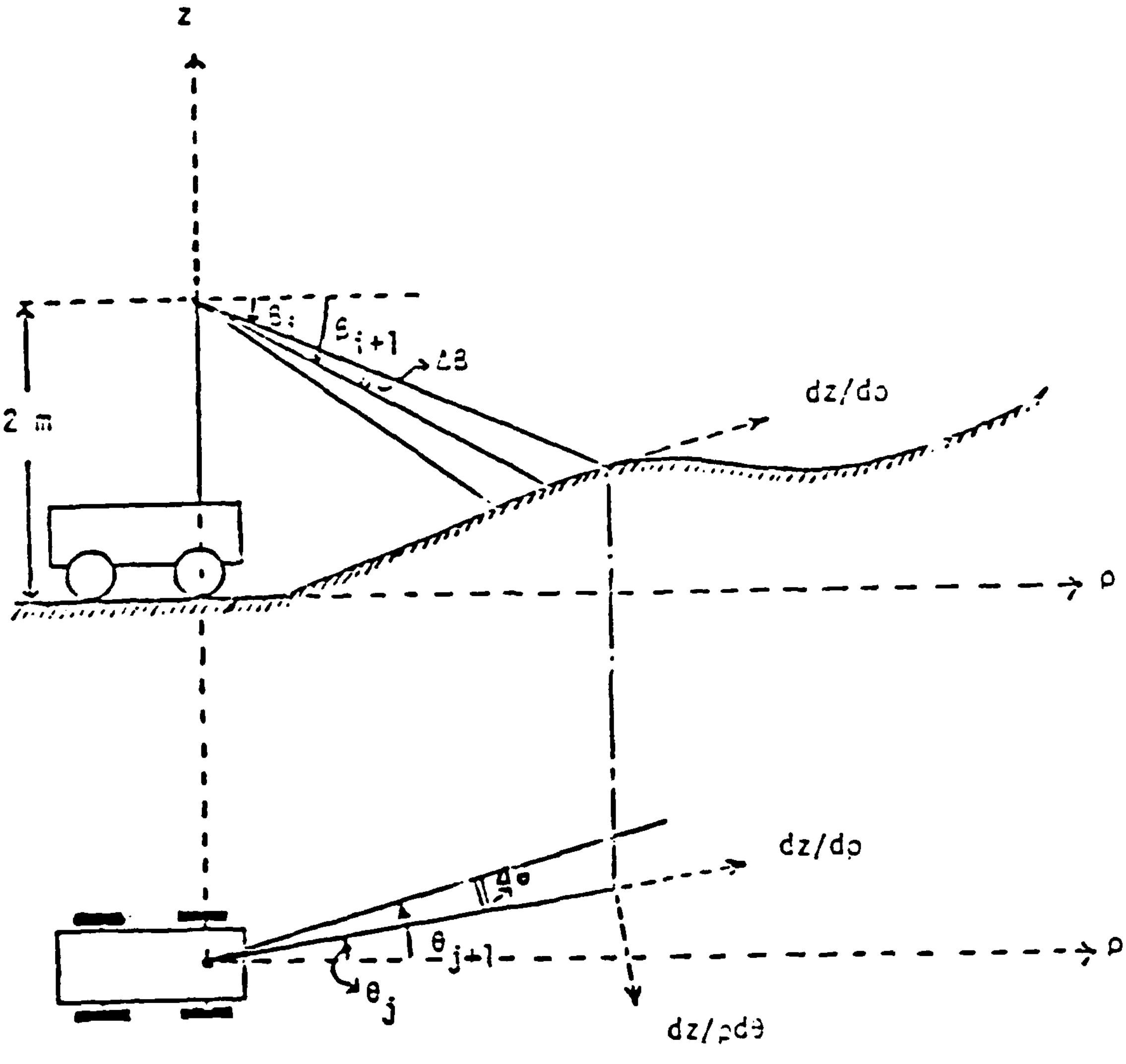

Figure 1. Top and side view of range finder.

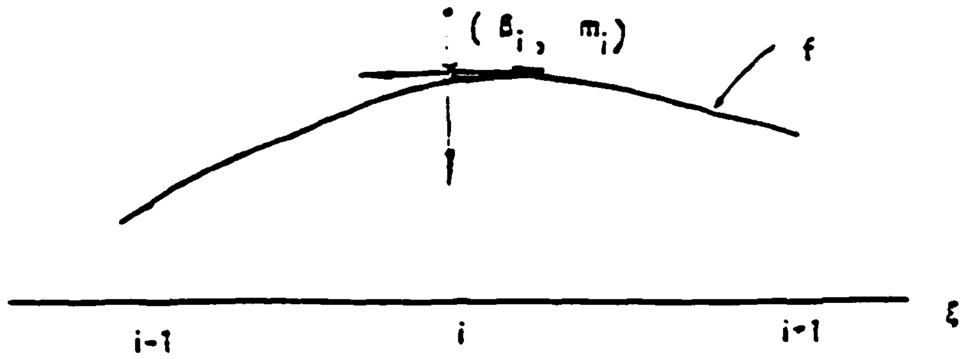

Figure 2. Random noise in angle and range measurements.

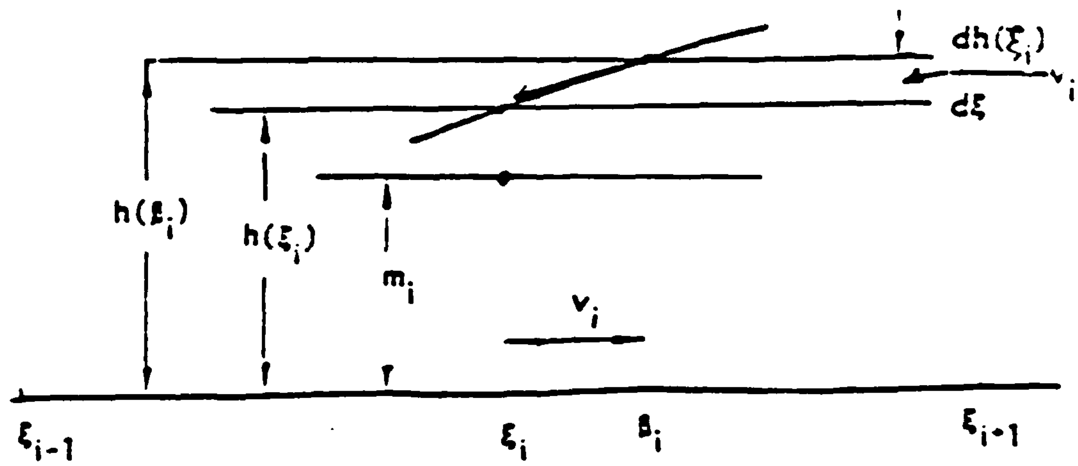

Figure 3. Representation of function due to angular errors.

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